

Decoherence in a nonequilibrium environment

Joseph Beer and Eric Lutz

Department of Physics, University of Augsburg, D-86135 Augsburg, Germany

(Dated: November 25, 2010)

We consider a quantum harmonic oscillator coupled to a general nonequilibrium environment. We show that the decoherence factor can be expressed in terms of a measurable effective temperature, defined via a generalized fluctuation-dissipation relation. We further propose a simple experimental scheme to determine the time-dependent effective temperature in a linear Paul trap with engineered reservoirs. Our formalism allows quantitative description of nonequilibrium decoherence in the presence of an arbitrary number of non-Markovian noise sources in a unified manner.

PACS numbers: 03.65.Yz

Recent theoretical and experimental work on decoherence has focused on the paradigmatic model of a single quantum harmonic oscillator (system) linearly coupled to an ensemble of harmonic oscillators (equilibrium reservoir) [1, 2]. The interaction with the reservoir leads to the dynamical suppression of interference fringes which arise in coherent superpositions of oscillator states. Decoherence, therefore, plays a fundamental role in the description of the quantum-to-classical transition [3, 4]. Consider for example a typical cat state, $\psi_{\text{cat}}(x) = [\psi_{-}(x) + \psi_{+}(x)]/\sqrt{2}$, consisting of a linear superposition of two Gaussian wave packets $\psi_{\pm}(x)$ of width σ and separated by a distance $d (\gg \sigma)$. Then, for temperatures much larger than the oscillator frequency, the interference-fringe contrast, or decoherence factor, decays exponentially with time according to [3, 4],

$$\mathcal{D}(t) = \exp(-D d^2 t) . \quad (1)$$

The coherence time $t_c = 1/(D d^2)$ is here inversely proportional to the amplitude D of the thermal fluctuations and to the square of the spatial separation of the wave packets. The Gaussian dependence of the decoherence factor on the distance d is responsible for the nonobservation of quantum superposition of macroscopic states. For an equilibrium reservoir, the coefficient D is related to the friction constant η via the Einstein relation, $D = \eta T$, where T is the temperature (we use natural units for which $\hbar = k_B = 1$). For a given coupling strength η , the decoherence rate is hence directly controlled by the temperature of the reservoir. The form of the equilibrium decoherence factor (1) has been successfully verified in experiments involving mesoscopic superpositions of a radiation field inside a microwave cavity [5] and of motional states of a single trapped ion [6]. In most cases of practical interest, however, a quantum system is subjected to many noise sources with different amplitudes and correlation times, corresponding de facto to a nonequilibrium environment.

In this paper we investigate nonequilibrium decoherence in the presence of many noise sources that act independently on the quantum system. The concrete system we have in mind is an ultracold ion confined in

a large scale rf trap and subjected to different fluctuating electric fields [10, 11], but our approach is more general. We model each noise source by an equilibrium harmonic reservoir with given coupling constant, temperature and correlation time, and take non-Markovian effects fully into account. Despite the complexity of the problem, we show that the nonequilibrium decoherence factor can be expressed in terms of a simple measurable quantity. To accomplish this task, we use recent results from nonequilibrium statistical physics and map the coupling to M different equilibrium reservoirs to the coupling to a single nonequilibrium reservoir that we characterize with the help of a time-dependent effective temperature. We define the latter via a nonequilibrium extension of the fluctuation-dissipation relation and suggest an experimental method to determine it in a linear Paul trap. The notion of effective temperature has been originally introduced in the description of glassy dynamics [8, 9]; it satisfies the expected properties of a temperature, such as a zeroth law (two interacting observables that evolve on the same time-scale have the same effective temperature) and has been measured in many nonequilibrium systems, for instance in classical spin [12] and colloidal [13] glasses, as well as in granular systems [14] and ageing polymer glasses [15]. We apply our results to the description of decoherence of a trapped ion interacting with two different amplitude reservoirs, treating the case of real and artificially engineered reservoirs [16]. We show that in both instances nonequilibrium decoherence can be slowed down in a controlled manner as compared to the corresponding equilibrium situation, and explain how this behavior can be understood and predicted with the help of the effective temperature.

The Hamiltonian of an oscillator S linearly coupled to M arbitrary harmonic reservoirs R_k is of the form

$$H = H_S + \sum_{k=1}^M \sum_{j=1}^{N_k} \frac{p_{kj}^2}{2m_{kj}} + \frac{m_{kj}\omega_{kj}^2}{2} \left(q_{kj} - \frac{c_{kj}x}{m_{kj}\omega_{kj}^2} \right)^2 , \quad (2)$$

where $H_S = p^2/(2m) + m\omega^2 x^2/2$ is the Hamiltonian of the system oscillator with the usual notation and c_{kj} denote coupling constants. Each equilibrium reservoir R_k

is fully specified by a temperature T_k and a spectral density function $J_k(\omega) = (\pi/2) \sum_j c_{kj}^2 / (m_{kj} \omega_{kj}) \delta(\omega - \omega_{kj})$ that characterizes the coupling to the system; we stress that we do not restrict ourselves to the Ohmic regime, $J_k(\omega) \sim \omega$, but include frequency-dependent damping. We describe the time evolution of the harmonic system with the exact quantum Langevin equation,

$$m\ddot{x}(t) + \int_0^t dt' \eta(t-t') \dot{x}(t') + m\omega^2 x(t) = F(t) - \eta(t)x(0), \quad (3)$$

which follows from the total Hamiltonian (2) [17]. Here the friction kernel $\eta(t) = \sum_k \eta_k(t)$ and the fluctuation force $F(t) = \sum_k F_k(t)$ are given by the sum of the respective contributions of the individual reservoirs. Under very general conditions (essentially that the reservoir is only weakly perturbed by the motion of the system [18]), any equilibrium reservoir can be modeled by an ensemble of harmonic oscillators [19]. Since all reservoirs R_k are in thermal equilibrium, the correlation functions $C_k(t) = \langle F_k(t)F_k(0) + F_k(0)F_k(t) \rangle / 2$ of the thermal forces and the friction kernels $\eta_k(t) = 2 \int_0^\infty d\omega J_k(\omega) / (\pi m \omega) \cos \omega t$ are connected via a fluctuation-dissipation relation. In the following, we consider the physically relevant regime for decoherence studies, in which temperature is larger than the oscillator frequency [5, 6]. The fluctuation-dissipation relation is then given in Laplace space by $C_k[s] = T_k \eta_k[s]$ [20], indicating that, at equilibrium, energy damping and thermal fluctuations evolve on the same time scale; we note that this equation can be interpreted as defining the equilibrium temperature T_k of each reservoir. The above relation only holds at thermal equilibrium. However, away from equilibrium, we can extend the fluctuation-dissipation relation and define a frequency-dependent effective temperature as [21],

$$T[s] = \frac{C[s]}{\eta[s]} = \frac{\sum_k T_k \eta_k[s]}{\sum_k \eta_k[s]}, \quad (4)$$

where we have introduced the correlation function $C[s] = \sum_k C_k[s]$ of the total fluctuating force $F[s]$. We mention that a low-temperature extension of the effective temperature has been proposed in Ref. [22]. With the help of Eq. (4), we can now map the complex problem of a quantum system coupled to M arbitrary equilibrium reservoirs to that of a system interacting with a single nonequilibrium reservoir characterized by a time-dependent effective temperature $T_{\text{eff}}(t) = \int_0^t dt' T(t')$ (see Fig. 1). In the case of a single equilibrium reservoir, $M = 1$, we obtain $T(t) = T_1 \delta(t)$ and the effective temperature reduces to the reservoir temperature, $T_{\text{eff}} = T_1$, as expected. Conversely, we note that a general nonequilibrium environment, with arbitrary $\eta(t)$ and $C(t)$, can be approximated with M equilibrated reservoirs [21]. The effective temperature is a useful quantity to characterize the violation of the fluctuation-dissipation relation in nonequilibrium systems. We will show below that it also

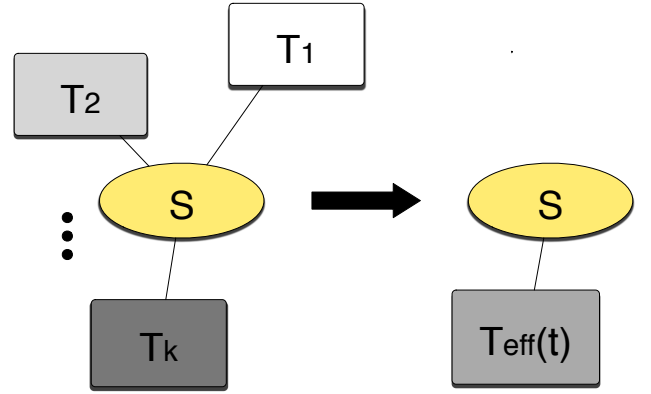


FIG. 1: Schematic representation of the open quantum system. A complex environment, made of different equilibrium reservoirs, is mapped to a single nonequilibrium reservoir, described by an effective time-dependent temperature.

provides deep insight into nonequilibrium decoherence.

We turn to the dynamics of decoherence of a cat state in a general nonequilibrium environment. We focus on the regime of microscopic separation that corresponds to long coherence times, and investigate the influence of a nonequilibrium reservoir which evolves on similar or shorter time scales. The importance of nonequilibrium effects on short time scales can be made clear by considering the coherent transport of ions in a segmented linear Paul trap: fast, nonadiabatic transport of the order of a few oscillation periods (i.e. a fraction of the coherence time) has been experimentally shown to drive ions into nonequilibrium excited states [23]. The accurate description of short-time nonequilibrium effects appears therefore crucial. Decoherence processes are conveniently analyzed in Wigner phase space [3]. The Wigner function of a Gaussian cat state is given by the sum of two classical contributions $W_{\pm}(x, p)$ and a quantum interference term $W_{\text{int}}(x, p)$, $W(x, p) = W_{-}(x, p) + W_{+}(x, p) + W_{\text{int}}(x, p)$. During decoherence, the interference term progressively decays as a function of time. We quantify the disappearance of the contrast of the interference fringes by means of the peak-to-peak ratio between the interference and the classical terms of the Wigner function [24],

$$\exp(-A_{\text{int}}) = \frac{W_{\text{int}}(x, p)|_{\text{peak}}}{2\sqrt{W_{-}(x, p)|_{\text{peak}}W_{+}(x, p)|_{\text{peak}}}}. \quad (5)$$

The nonequilibrium peak-to-peak ratio can be directly expressed as a function of the effective temperature (4) in the following way. We first note that for high temperatures, the dynamics of the system is dominated by momentum diffusion driven by the fluctuating force $F(t)$. We can thus describe the evolution of the Wigner function by the approximate Fokker-Planck equation, where

free evolution and dissipation terms are neglected [3],

$$\frac{\partial W(x, p, t)}{\partial t} = \eta(t)x \frac{\partial W}{\partial p} + \sigma_{pp}^2(t) \frac{\partial^2 W}{\partial p^2}. \quad (6)$$

Here $\sigma_{pp}^2(t) = 2 \int_0^t dt' \int_0^{t'} dt'' C(t' - t'')$ is the momentum variance which describes the width of the Wigner function. The attenuation factor can now be readily evaluated from the solution of Eq. (6) and we find $A_{\text{int}}(t) = d^2 \sigma_{pp}^2(t)/2$. According to the definition (4) of the effective temperature, the noise correlator is further given by the convolution $C(t) = \int_0^t dt' \eta(t - t') T(t')$. We therefore obtain,

$$A_{\text{int}}(t) = d^2 \int_0^t dt' \int_0^{t'} dt'' \eta(t' - t'') T(t'') (t - t'). \quad (7)$$

The above nonequilibrium peak-to-peak ratio $\exp(-A_{\text{int}})$ is the direct extension of the familiar equilibrium decoherence factor (1) to which it reduces when $T(t) = T\delta(t)$; it completely characterizes decoherence of a quantum system coupled to a general nonequilibrium environment, with arbitrary $\eta(t)$ and $C(t)$. As in the equilibrium case, for a given coupling strength the decoherence rate is determined by the (effective) temperature of the nonequilibrium reservoir.

Let us now illustrate our theoretical results by analyzing the concrete situation of a quantum oscillator that interacts with a fast and a slow reservoir. This is the simplest non-trivial out-of-equilibrium environment [21]. We begin by discussing 'real' environments before treating the case of 'engineered' environments in ion traps in the next section. We specify the fast (Markovian) reservoir by a temperature T_f and a delta-correlated friction kernel, $\eta_f(t) = \eta_f \delta(t)$, and the slow (non-Markovian) reservoir by a temperature T_s and an exponentially-correlated kernel, $\eta_s(t) = (\eta_s/\tau) \exp(-t/\tau)$, with correlation time τ . The effective temperature (4) is then given by

$$T[s] = \frac{T_f \eta_f (s\tau + 1) + T_s \eta_s}{\eta_f (s\tau + 1) + \eta_s}, \quad (8)$$

and the corresponding effective temperature in the time-domain reads, with $\eta = \eta_f + \eta_s$,

$$T_{\text{eff}}(t) = \frac{\eta_f T_f + \eta_s T_s}{\eta} + \frac{\eta_s}{\eta} (T_f - T_s) \exp\left(-\frac{\eta}{\eta_f} \frac{t}{\tau}\right). \quad (9)$$

The effective temperature (9) evolves monotonously from the fast reservoir temperature T_f at $t = 0$ to the weighted average $T_\infty = (\eta_f T_f + \eta_s T_s)/\eta$ of the two temperatures at large times, $t \gg (\eta_f/\eta)\tau$. The exact peak-to-peak ratio $\exp(-A_{\text{int}})$ (5) for the nonequilibrium two-reservoir model is plotted in Fig. 2 for three different sets of reservoir parameters. We observe that the pace of decoherence can be controlled by properly tuning the reservoir variables: nonequilibrium decoherence is slower

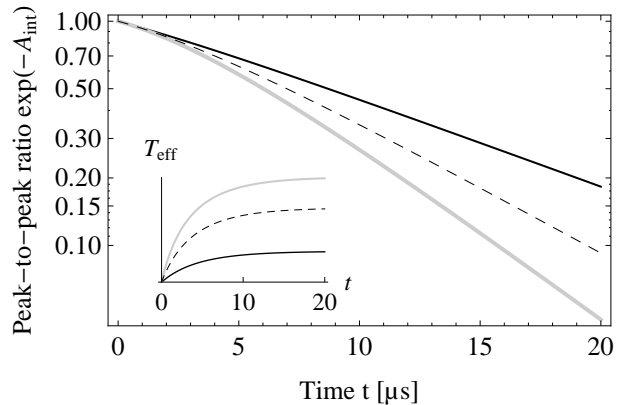


FIG. 2: Interference-fringe contrast for a nonequilibrium two-reservoir system simulated by two random electric fields $E_1(t)$ and $E_2(t)$. Nonequilibrium decoherence is slower (grey) or faster (black) than equilibrium decoherence (dashed) when the temperature $T_f \sim \langle E_1^2 \rangle$ of the fast reservoir is higher or lower than the temperature $T_s \sim \langle E_2^2 \rangle$ of the slow reservoir. The dashed curve is the reference line corresponding to equal noise amplitudes $\langle E_1^2 \rangle = \langle E_2^2 \rangle$. The time-dependent effective temperature (shown in the inset) predicts this behavior: it is smaller than the reference temperature in the first case, while it is larger in the second. The effective temperature can be determined from measurements of the motional heating rate for trap frequencies $\omega/(2\pi)$ varying between 1MHz and 100MHz. Parameters are: $d = 7\text{nm}$, $\sigma = 0.12\text{nm}$, $\tau = 5\mu\text{s}$, $\langle E_1^2 \rangle = 10^{-9}\text{V}^2\text{m}^{-2}$ and $\langle E_2^2 \rangle = 6 \cdot 10^{-10}\text{V}^2\text{m}^{-2}$.

(faster) than equilibrium decoherence, which corresponds to $T_f = T_s$, when the temperature of the slow reservoir is lower (higher) than the temperature of the fast reservoir. We can therefore dramatically affect the decoherence properties of the system by, for instance, simply exchanging the correlation times of the two reservoirs, while keeping their respective temperatures constant. This is a nontrivial non-Markovian effect. The effective temperature T_{eff} offers an intuitive explanation for this behavior. Following Eq. (9), we note that when $T_f > T_s$ ($T_f < T_s$), the two-reservoir configuration is equivalent to a nonequilibrium reservoir with a decreasing (increasing) effective temperature. The approximate expression (7) of the interference-fringe contrast then immediately shows that decoherence is slowed down (or accelerated).

Linear Paul traps are promising candidates for quantum computation and an invaluable tool for the investigation of the dynamics of decoherence [25]. Due to their weak interaction with the ambient reservoir, and the resulting long coherence time, they offer the unique possibility of exploring various decoherence scenarios generated by the coupling to well-defined engineered reservoirs [16]. The coupling to an equilibrium amplitude reservoir can thus be simulated with the help of a fluctuating electric field that acts on the charged ion [6]. In this case, the temperature of the reservoir is given by the variance of the field. The nonequilib-

rium two-reservoir configuration can be effectively simulated with two random electric fields $E_1(t)$ and $E_2(t)$ with corresponding correlation times. The correlation function of the superimposed noisy fields is given by $C^E(t) = e^2 \langle E_1^2 \rangle \delta(t) + (e^2/\tau) \langle E_2^2 \rangle \exp(-t/\tau)$ and is identical to the two-reservoir correlation function $C(t)$ with the identification $e^2 \langle E_1^2 \rangle = \eta_f T_f$ and $e^2 \langle E_2^2 \rangle = \eta_s T_s$. Using the definition (4) of the effective temperature, we then find $T^E[s] = T + C^E[s]/\eta^E[s] \simeq C^E[s]/\eta^E[s]$, since the influence of the ambient reservoir can be neglected during the duration of the experiment. We note that the damping kernel $\eta^E[s] = \eta$ is here constant, in contrast to the true two-reservoir case. As a result, the effective temperature for the two electric fields is $T_{\text{eff}}^E(t) = (e^2/\eta) (\langle E_1^2 \rangle + \langle E_2^2 \rangle (1 - \exp(-t/\tau)))$, as shown in Fig. 2 (inset) for realistic trap parameters. Again, the decoherence properties can be understood with the help of the effective temperature: faster (slower) decoherence corresponds to higher (lower) effective temperature, as compared to the equal amplitude $\langle E_1^2 \rangle = \langle E_2^2 \rangle$ situation. The fact that $T_{\text{eff}}^E(t)$ increases in the latter indicates that the noisy fields heat the system up, a direct consequence of the non-Markovian nature of the environment (T_{eff}^E is a constant in the limit $\tau \rightarrow 0$). We also mention that the exact (5) and approximate (7) peak-to-peak ratios are indistinguishable for the parameters of Fig. 2. We can now readily generalize these results to arbitrary complex situations: Given an unknown number of noise sources, with unknown amplitudes and correlation times, we can use the measurable effective temperature to predict the decoherence rate of the system via formula (7).

An important property of the effective temperature is that it can be measured [12–15]. We next discuss an experimental scheme that allows its determination in ion traps. We consider a single ion coupled to a nonequilibrium amplitude reservoir generated by an unspecified number of electric fields with different noise amplitudes and correlation times (noisy electrical electrode potentials are the main sources of trap decoherence [11]). The corresponding effective temperature (4) is given by the ratio of the noise correlation function and the friction kernel. Both quantities can be determined experimentally by measuring the motional heating rate of the trap [26]. The latter is related to the power spectrum of the fluctuation force, $S(\omega) = \int dt \exp(i\omega t) C(t)$, by $\dot{n} \simeq e^2 S(\omega)/(4m\hbar\omega)$, where e is the charge of the ion [26]. The noise correlation function can hence be directly obtained from measurements of the heating rate for different trap frequencies ω . A first set of measurements of the heating rate for the ambient equilibrium reservoir alone yields the friction kernel η (which in this case has been shown to be frequency independent [26]) via the fluctuation-dissipation relation, $S(\omega) = 2D = 2\eta T$. A second set of measurements of the heating rate in the presence of the random electric fields further provides the correlation function $C(t)$ of the nonequilibrium engi-

neered reservoir and, in turn, the time-dependent effective temperature.

To conclude, the present findings show that nonequilibrium decoherence induced by an unknown number of different noise sources can be successfully described within a unified framework, in a way similar to equilibrium decoherence. They further highlight the central role of the effective temperature to characterize, understand and predict the loss of coherence of a quantum system in a general nonequilibrium environment.

We thank G. Huber and F. Schmidt-Kaler for useful discussions. This work was supported by the Emmy Noether Program of the DFG (LU1382/1-1) and the cluster of excellence Nanosystems Initiative Munich (NIM).

-
- [1] D. Giulini *et al.*, *Decoherence and the Appearance of a Classical World in Quantum Theory*, (Springer, Berlin, 1996).
 - [2] M.A. Schlosshauer, *Decoherence and the Quantum-To-Classical Transition*, (Springer, Berlin, 2007).
 - [3] W. H. Zurek, Phys. Today **44**, 3644 (1991).
 - [4] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
 - [5] M. Brune *et al.*, Phys. Rev. Lett. **77**, 4887 (1996).
 - [6] C. J. Myatt *et al.*, Nature **403**, 269 (2000).
 - [7] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge, Cambridge, 2000).
 - [8] L. F. Cugliandolo and J. Kurchan, Phys. Rev. Lett. **71**, 173 (1993).
 - [9] L. F. Cugliandolo, J. Kurchan and L. Peliti, Phys. Rev. E **55**, 3898 (1997).
 - [10] D. Kielpinski, C. Monroe and D. J. Wineland, Nature **417**, 709 (2002).
 - [11] L. Deslauriers *et al.*, Phys. Rev. Lett. **97**, 103007 (2006).
 - [12] D. Hérissin and M. Ocio, Phys. Rev. Lett. **88**, 257202 (2002).
 - [13] B. Abou and F. Gallet, Phys. Rev. Lett. **93**, 160603 (2004).
 - [14] A. R. Abate and D. J. Durian, Phys. Rev. Lett. **101**, 245701 (2008).
 - [15] H. Oukris and N. E. Israeloff, Nature Phys. **6**, 135 (2010).
 - [16] J. F. Poyatos, J. I. Cirac and P. Zoller, Phys. Rev. Lett. **77**, 4728 (1996).
 - [17] U. Weiss, *Quantum Dissipative Systems*, (World Scientific, Singapore, 1993).
 - [18] A.O. Caldeira and A.J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
 - [19] P. Reimann, Chem. Phys. **268**, 337 (2001).
 - [20] H. B. Callen and J. A. Welton, Phys. Rev. **83**, 34 (1951).
 - [21] F. Zamponi *et al.*, J. Stat. Mech.: Theory and Experiment, **P09013**, (2005).
 - [22] L. F. Cugliandolo and G. Lozano, Phys. Rev. Lett. **80**, 4979 (1998).
 - [23] G. Huber *et al.*, New J. Phys. **10**, 013004 (2008).
 - [24] J. P. Paz, S. Habib and W. H. Zurek, Phys. Rev. D **47**, 488 (1993).
 - [25] D. Leibfried *et al.*, Rev. Mod. Phys. **75**, 281 (2003).
 - [26] Q. A. Turchette *et al.*, Phys. Rev. A **62**, 053807 (2000).